Homework 10

MATH 231

Due Wednesday, November 26, 2025

Instructions. *In lieu of a quiz, you will submit your hand-written solution to Exercise 2 either in class on Wednesday, November 26 or a scanned copy via email (nvlamis@qc.cuny.edu).*

Exercise 1. Complete the following exercises from Section 5.3 in the course textbook:

1,3, 5, 7, 11, 29, 30, 31, 33

Exercise 2. Pictured to the right is a Markov chain.

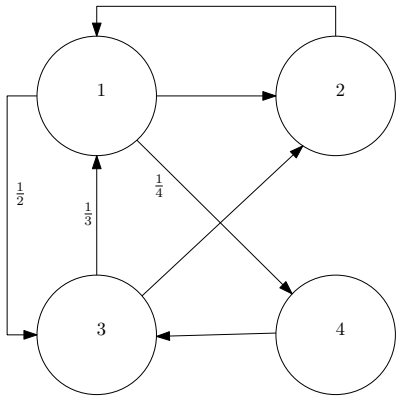
(a) Fill in the missing edge labels.

(b) Write down the transition matrix M for the Markov chain.

(c) Find the unique stochastic vector $\mathbf{w}$ such that

$$M\mathbf{w} = \mathbf{w}.$$

(Note that the Markov chain is regular, so you know that $\mathbf{w}$ exists.)



Exercise 3. Consider the (incomplete) matrix:

$$A = \begin{bmatrix} \frac{1}{3} & * & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & * \\ * & \frac{2}{7} & 0 & * & 0 \\ 0 & \frac{3}{7} & 0 & 0 & \frac{1}{5} \\ \frac{1}{6} & 0 & * & 0 & 0 \end{bmatrix}$$

(a) Replace each asterisk mark in A with a real number so that the result is a stochastic matrix.

(b) Draw the Markov chain associated to A.

(c) Determine if the underlying directed graph is strongly connected.

Exercise 4. Draw a Markov chain representing the following situation: *The weather in Edinburgh is either good, indifferent, or bad on any given day. If the weather is good today, there is a 50% chance the weather will be good tomorrow, a 30% chance the weather will be indifferent, and a 20% chance the weather will be bad. If the weather is indifferent today, it will be good tomorrow with probability .20 and indifferent with probability .70. Finally, if the weather is bad today, it will be good tomorrow with probability .10 and indifferent with probability .30.*

Exercise 5. Let M be an $n \times n$ stochastic matrix, and let $\mathbf{w} \in \mathbb{R}^n$ be a stochastic vector. Show that $M\mathbf{w}$ is stochastic.