Homework 12

MATH 231

Due Wednesday, December 10, 2025

Instructions. *We will have a quiz in class on the due date based on the content from the assignment. See the back of the textbook for solutions and hints for odd-numbered problems.*

Exercise 1. Complete the following exercises from Section 6.1 in the course textbook:

38, 39

Exercise 2. Complete the following exercises from Section 6.2 in the course textbook:

41

Exercise 3. Let $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and let $W = \text{span}\{\mathbf{v}\}$. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(\mathbf{u}) = \text{proj}_W(\mathbf{u})$$

(In #41 in Section 6.2, you established that T is a linear transformation.) Find the matrix A satisfying $T(\mathbf{u}) = A\mathbf{u}$ for every $\mathbf{u} \in \mathbb{R}^2$.

Exercise 4. Complete the following exercises from Section 6.3 in the course textbook:

1, 3, 5, 7, 31, 32

Exercise 5. Let W be a subspace of $\mathbb{R}^n$.

(a) Show that $W \subset (W^\perp)^\perp$.

(b) Show that $(W^\perp)^\perp \subset W$. (Hint: Let $w \in (W^\perp)^\perp$). Use the orthongal projection theorem to write $w = \widehat{w} + z$, where $\widehat{w} \in W$ and $z \in W^\perp$. Argue that $z = \mathbf{0}$.)

Together, parts (a) and (b) show that $W = (W^\perp)^\perp$. (Fun fact: this fails in infinite dimensional spaces!)

Exercise 6. This exercise gives another proof of the fact that $W = (W^\perp)^\perp$).

(a) Let X and Y be subspaces of $\mathbb{R}^n$ with $Y \subset X$. Show that if $\dim X = \dim Y$, then $X = Y$.

(b) Combine part (a) of this exercise together with part (a) from Exercise 5 and part (c) of Exercise 32 in Section 6.3 to deduce that if W is a subspace of $\mathbb{R}^n$, then $W = (W^\perp)^\perp$.