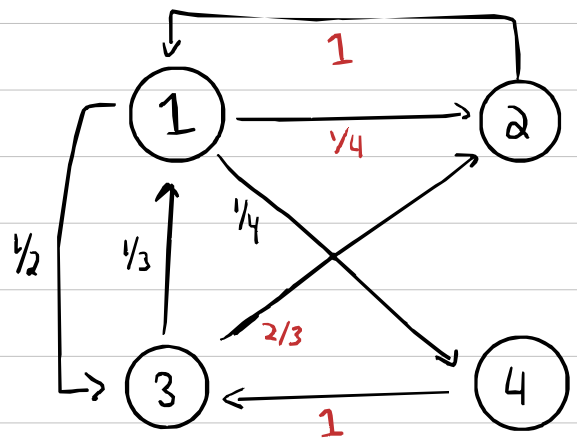
Quiz 10 Solutions

(For Quiz 10, you were asked to submit a complete solution to Exercise 2 on HW 10.)

(a) The completed Markov chain:



(Added values in red.)

(b)

$$M = \begin{bmatrix} 0 & 1 & 1/3 & 0 \\ 1/4 & 0 & 2/3 & 0 \\ 1/2 & 0 & 0 & 1 \\ 1/4 & 0 & 0 & 0 \end{bmatrix}$$

(c) Solve $Mx = x$. $Mx = x \iff (M-I)x = 0$

$$M-I = \begin{bmatrix} -1 & 1 & 1/3 & 0 \\ 1/4 & -1 & 2/3 & 0 \\ 1/2 & 0 & -1 & 1 \\ 1/4 & 0 & 0 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & -1/3 & 0 \\ 0 & -3/4 & 3/4 & 0 \\ 0 & 1/2 & -5/6 & 1 \\ 0 & 1/4 & 1/12 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1/2 & -5/6 & 1 \\ 0 & 1/4 & 1/12 & -1 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & -1 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1/3 & 1 \\ 0 & 0 & 1/3 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution to $Mx = x$:

$$\left.\begin{aligned} x_1 &= 4t \\ x_2 &= 3t \\ x_3 &= 3t \\ x_4 &= t \end{aligned}\right\} \Rightarrow \text{Solution set spanned by } \begin{bmatrix} 4 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

Now, $4+3+3+1 = 11$

$\Rightarrow w = \frac{1}{11}\begin{bmatrix} 4 \\ 3 \\ 3 \\ 1 \end{bmatrix}$ is the stochastic vector

satisfying $Mw = w$