Solutions

Quiz 11
Wednesday, December 3, 2025

MATH 231
Fall 2025

Problem 1. Let $\mathbf{x} = \begin{bmatrix} 10 \\ -3 \end{bmatrix}$ and let $\mathbf{y} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$. Compute the following:

(a) $\mathbf{x} \cdot \mathbf{y}$

2 $= (10)(-1) + (-3)(-5)$

$= -10 + 15$

$= 5$

(b) the norm of $\mathbf{y}$ (that is, $\|\mathbf{y}\|$)

2 $\|y\| = \sqrt{y \cdot y} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$

(c) the distance between $\mathbf{x}$ and $\mathbf{y}$.

2 distance $= \|x - y\| = \left\| \begin{bmatrix} 11 \\ 2 \end{bmatrix} \right\| = \sqrt{11^2 + 2^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$

Problem 2. Let $W = \text{span}\{\mathbf{u}, \mathbf{v}\}$. Show that if $\mathbf{x}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{x}$ is orthogonal to every vector in W.

Let $z \in \text{span}\{u, v\}$.

4 $\Rightarrow z = c_1 u + c_2 v$ for some $c_1, c_2 \in \mathbb{R}$

$\Rightarrow x \cdot z = x \cdot (c_1 u + c_2 v) = x \cdot (c_1 u) + x \cdot c_2 v$

$= c_1 (x \cdot u) + c_2 (x \cdot v)$

$= c_1 \cdot 0 + c_2 \cdot 0$

$= 0$

$\Rightarrow x$ is orthogonal to z