Solutions

Quiz 12

Wednesday, December 10, 2025

MATH 231
Fall 2025

Problem 1. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$.

(a) Verify that $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal.

$$u_1 \cdot u_2 = (1)(5) + (3)(1) + (-2)(4) = 5 + 3 - 8 = 0$$

(b) Let $\mathbf{y} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ and let $W = \operatorname{span}\{\mathbf{u}_1, \mathbf{u}_2\}$. Write $\mathbf{y}$ as a sum of a vector in W and a vector orthogonal to W.

Let $\hat{y} = \operatorname{proj}_W(y) \in W$

$$\Rightarrow \hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2 = 0 \cdot u_1 + \frac{28}{42} u_2 = \frac{2}{3}\begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$$

Let $z = y - \hat{y} \in W^\perp$

$$\Rightarrow z = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -7 \\ 7 \\ 7 \end{bmatrix} = \frac{7}{3}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Now, $y = \hat{y} + z$ w/ $\hat{y} \in W$ and $z \in W^\perp$

Problem 2. Let W be a subspace of $\mathbb{R}^n$. Show that if $\mathbf{v}$ is in both W and $W^\perp$, then $\mathbf{v} = 0$.

Let $v \in W \cap W^\perp$.

Since $v \in W$ and $v \in W^\perp$, we must have that

$$v \cdot v = 0.$$

So, $\|v\| = 0$, implying $v = 0$.