Quiz 2 MATH 231

Wednesday, September 17, 2025 Fall 2025

Problem 1. Given below is the reduced row echelon form of an augmented matrix associated to a linear system in the variables x_1, x_2, x_3, and x_4.

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) List the basic variable(s).

$x_1, x_2,$ and x_4

(b) List the free variable(s).

x_3

(c) Write down the general solution to the linear system.

$x_1 = -3t + 5$

$x_2 = -2t - 3$

$x_3 = t$

$x_4 = 0$

(d) How many solutions does the linear system have?

infinitely many

(Turn Page Over)

Problem 2. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) Write down the linear system corresponding to the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{b}.$$

$$\begin{aligned} x_1 + 3x_2 &= 0 \\ x_1 + x_2 &= 1 \\ 6x_2 &= 0 \end{aligned}$$

(b) Write the linear system you gave in part (a) as a matrix equation.

$$\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 0 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(c) Is **b** a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$? Justify your answer.

No:

b is a linear combination of v_1 and v_2 only if the linear system from (a) has a solution. But we see that $x_2 = 0$ from the third equation, implying that $x_1 = 0$ from plugging $x_2 = 0$ into the first equation. Now, the second equation then says that $0 + 0 = 1$, which is false!