Solutions

Quiz 3 MATH 231

Wednesday, October 6, 2025 Fall 2025

Problem 1. Let $A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$.

(a) Compute rref(A).

$$\begin{bmatrix} 1 & 0 & 9 & -8 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

(b) Does $A\mathbf{x} = \mathbf{b}$ have a solution for every $b \in \mathbb{R}^2$? Explain.

Yes, as every row of A contains a pivot position.

As a consequence, in the augmented matrix for $Ax=b$, the rightmost column cannot be a pivot column, implying $Ax=b$ is consistent.

(c) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form (or as a span of vectors).

$$\left.\begin{aligned} x_1 &= -9s + 8t \\ x_2 &= 4s - 5t \\ x_3 &= s \\ x_4 &= t \end{aligned}\right\} \qquad \left\{ \begin{bmatrix} -9s+8t \\ 4s-5t \\ s \\ t \end{bmatrix} \;\middle|\; s,t \in \mathbb{R} \right\} = \text{Span}\left\{ \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Turn page over.

Problem 2. Given $A = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}$, observe that the third column is the sum of the first two columns. Find a nontrivial solution of $A\mathbf{x} = \mathbf{0}$. (No computations are necessary.)

$$x = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Where does this come from?

$$A = [\,\vec{a}_1 \;\; \vec{a}_2 \;\; \vec{a}_3\,] \quad \text{and} \quad \vec{a}_3 = \vec{a}_1 + \vec{a}_2$$

$$\text{Now,} \quad A\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \vec{a}_1 + \vec{a}_2 - \vec{a}_3 = \vec{a}_3 - \vec{a}_3 = \vec{0}.$$