Quiz 5 MATH 231
Wednesday, October 22, 2025 Fall 2025

Problem 1. Suppose A and B are $n \times n$ matrices such that both AB and B are invertible. Show that A is invertible.

B is invertible $\Rightarrow B^{-1}$ is invertible

$\Rightarrow (AB)B^{-1}$ is invertible, as both AB and B^{-1} are invertible

Now $(AB)B^{-1} = A(BB^{-1}) = AI = A$

$\Rightarrow A$ is invertible

Problem 2. If A, B, and C are invertible $n \times n$ matrices, does the equation

$$C^{-1}(A + X)B^{-1} = I_n$$

have a solution? If so, finid it.

$$C^{-1}(A+X)B^{-1} = I_n$$

$$\Rightarrow \quad C\left(C^{-1}(A+X)B^{-1}\right)B = C I_n B$$

$$\Rightarrow \quad A + X = CB$$

$$\Rightarrow \quad X = CB - A \quad \text{is a solution}$$