Quiz 6
Wednesday, October 29, 2025

MATH 231
Fall 2025

Problem 1. Use the algorithm from class to determe in the matrix A below is invertible, and if it is, find its inverse.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

We need to row reduce the following matrix $[A\ I_3]$:

$$[A\ I_3] = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{bmatrix} \rightsquigarrow$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix} \rightsquigarrow$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{bmatrix} = [I_3\ A^{-1}] \Rightarrow A \text{ is invertible w/}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 16 & 6 & 2 \\ 20 & 8 & 2 \\ 7 & 3 & 1 \end{bmatrix}$$

Problem 2. Can a square matrix with two identical columns be invertible? Why or why not?

Told everyone to skip. It wasn't clear what I wanted as a solution. I give a complete solution below.

Let $A = [\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_n]$ be a square matrix. Recall that if A is invertible, then $A\vec{x} = \vec{0}$ has only the trivial solution. Indeed, if $A\vec{v} = \vec{0}$, then $\vec{v} = A^{-1}(A\vec{v}) = A^{-1}\vec{0} = \vec{0}$.

Now, if $\vec{a}_i = \vec{a}_j$ for some $i \neq j$, then $A(\vec{e}_i - \vec{e}_j) = \vec{a}_i - \vec{a}_j = \vec{0}$.

$\Rightarrow A\vec{x} = \vec{0}$ has a nontrivial solution $\Rightarrow A$ is not invertible. $\square$