

Solutions

Quiz 7

Wednesday, November 5, 2025

MATH 231

Fall 2025

Problem 1. Below is a matrix A and an echelon form of A .

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A .

The first two columns of A are pivot columns.

$$\Rightarrow \left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\} \text{ is a basis for } \text{col } A$$

(b) Find a basis for the null space of A .

Let's go ~~than~~ one step further to get $\text{rref}(A)$.

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -4 & 7 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Free variables in $Ax=0$ are x_3 and x_4 .

General solutions:

$$\left. \begin{array}{l} x_1 = 4s - 7t \\ x_2 = -5s + 6t \\ x_3 = s \\ x_4 = t \end{array} \right\} \Rightarrow \text{null } A = \text{span} \left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}$$

These two vectors give a basis for $\text{null}(A)$.

(c) What are the dimensions of the null space and of the column space of A ?

$$\dim \text{col } A = 2$$

$$\dim \text{null } A = 2$$

(Turn page over.)

Problem 2. If the subspace of all solutions of $Ax = 0$ has a basis consisting of three vectors and if A is a 5×7 matrix, what is the rank of A ? Explain your thought process.

Given that $\dim \text{null}(A) = 3$.

The rank-nullity theorem now tells us that

$$\text{rank}(A) + 3 = 7, \text{ implying that}$$

$$\text{rank}(A) = 4.$$